UNIVERSITY OF TORONTO SCARBOROUGH

Department of Computer and Mathematical Sciences

STAC67H3F: Regression Analysis

Quiz 2: written part

TUT0001 · Tuesday November 3, 2026

Instructions

- **Quiz 2 has two parts.** This **written part** (25 minutes, 12 points) comes first, on paper. The **Quercus part** (25 minutes, 8 points) follows, on your laptop, once this paper has been collected. Quiz 2 is marked out of 20.
- **Keep your laptop closed and your phone away** until this paper has been collected. A non-programmable calculator is permitted; no other aids.
- Answer in the space provided on pages 2 and 3. This booklet is scanned: anything written elsewhere is not marked. **Do not write on the QR codes** at the top of the pages.
- Show your work: a number with no supporting work receives no credit.
- Round *final* answers to 2 decimal places; do not round intermediate quantities.

Good luck!

1. A horticulturist grows tomatoes in $n = 25$ greenhouse plots and records, for each plot, the **yield** Y (in kg), the amount of **fertilizer** x_1 (in g per m^2, from 3 to 10) and the amount of **irrigation** x_2 (in litres per day, from 3 to 12). The model is $Y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \varepsilon_i$, with $\varepsilon_1, \ldots, \varepsilon_{25}$ independent $N(0, \sigma^2)$. With $\boldsymbol{X}$ the 25×3 design matrix (its columns are a column of ones, x_1 and x_2), the following results are obtained:

$$(\boldsymbol{X}'\boldsymbol{X})^{-1} = \begin{bmatrix} 0.74 & -0.05 & -0.05 \\ -0.05 & 0.015 & -0.005 \\ -0.05 & -0.005 & 0.01 \end{bmatrix}, \qquad \hat{\boldsymbol{\beta}} = \begin{bmatrix} 10 \\ 1.2 \\ 0.6 \end{bmatrix}, \qquad SSE = 88.$$

   (a) Interpret $\hat{\beta}_1$ in the context of the study. One of the plots received 4 g per m^2 of fertilizer and 6 litres per day of irrigation, and yielded 20.1 kg: give its fitted value and its residual. [3]

Quantiles of the Student distribution: $t(p;\,\nu)$ is the p quantile of $t(\nu)$.

ν	$t(0.90;\,\nu)$	$t(0.95;\,\nu)$	$t(0.975;\,\nu)$	$t(0.99;\,\nu)$
15	1.341	1.753	2.131	2.602
16	1.337	1.746	2.120	2.583
17	1.333	1.740	2.110	2.567
18	1.330	1.734	2.101	2.552
19	1.328	1.729	2.093	2.539
20	1.325	1.725	2.086	2.528
21	1.323	1.721	2.080	2.518
22	1.321	1.717	2.074	2.508
23	1.319	1.714	2.069	2.500
24	1.318	1.711	2.064	2.492

(b) Is irrigation related to the mean yield once fertilizer is taken into account? Answer with a t test at the 5% level: state the hypotheses, compute the test statistic, give the critical value with its degrees of freedom, and conclude in context. [5]

(c) Give a 95% confidence interval for the mean yield of plots that receive 8 g per m^2 of fertilizer and 9 litres per day of irrigation. [4]