STAC67H3F: Quiz 1, written part: solutions

TUT0001 (Tue 29 Sep) and TUT0002 (Wed 30 Sep), Fall 2026

Version A: TUT0001, rent against floor area

Given $n = 24$, $\sum x_i = 1444.2$, $\sum Y_i = 31854.0$, $S_{xx} = 3288.2$, $S_{YY} = 1004896.3$, $S_{xY} = 51465.3$.

(a) Estimates and fitted line

$\bar{x} = 60.175$ and $\bar{Y} = 1327.25$.

$$\hat{\beta}_1 = \frac{S_{xY}}{S_{xx}} = \frac{51465.3}{3288.2} = 15.6515, \qquad \hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{x} = 1327.25 - 15.6515 \times 60.175 = 385.42$$

Fitted line: $\hat{Y} = 385.42 + 15.65\, x$.

(b) Interpretation

Each additional square metre of floor area is associated with an increase of about **$15.65 in mean monthly rent**.

(c) Show that $SSE = S_{YY} - S_{xY}^2 / S_{xx}$

Replace $\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{x}$ in the residual:

$$e_i = Y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i = (Y_i - \bar{Y}) - \hat{\beta}_1 (x_i - \bar{x}).$$

Square and sum:

$$SSE = \sum_{i=1}^{n} e_i^2 = S_{YY} - 2\hat{\beta}_1 S_{xY} + \hat{\beta}_1^2 S_{xx}.$$

Substitute $\hat{\beta}_1 = S_{xY}/S_{xx}$:

$$SSE = S_{YY} - 2\,\frac{S_{xY}^2}{S_{xx}} + \frac{S_{xY}^2}{S_{xx}} = S_{YY} - \frac{S_{xY}^2}{S_{xx}}.$$

Version B: TUT0002, delivery time against distance

Read from the output: $\hat{\beta}_1 = 1.900$, $SE(\hat{\beta}_1) = 0.182$, $n = 22$; and, in part (c), the unbiased sample variance $s_x^2 = 15.75$, i.e. with divisor $n - 1$.

(a) Interpretation

Each additional kilometre is associated with an increase of about **1.90 minutes in mean delivery time**.

(b) A 97% confidence interval for β_1

$\alpha = 0.03$, so the quantile is $t(1-\alpha/2;\, n-2) = t(0.985;\, 20) = 2.336$.

$$\hat{\beta}_1 \pm t(0.985;\, 20)\, SE(\hat{\beta}_1) = 1.900 \pm 2.336 \times 0.182 = 1.900 \pm 0.4252 = \big(1.47,\ 2.33\big).$$

(c) $\hat{\sigma}^2$ from s_x^2

$SE(\hat{\beta}_1) = \hat{\sigma}/\sqrt{S_{xx}}$, and $S_{xx} = \sum(x_i - \bar{x})^2 = (n-1)\, s_x^2$. So

$$S_{xx} = 21 \times 15.75 = 330.75, \qquad \hat{\sigma}^2 = SE(\hat{\beta}_1)^2\, S_{xx} = 0.182^2 \times 330.75 = 10.96.$$

(d) Larger S_{xx}

Narrower. $SE(\hat{\beta}_1) = \hat{\sigma}/\sqrt{S_{xx}}$, so with $\hat{\sigma}$ and n (hence the quantile) unchanged, a larger S_{xx} gives a smaller standard error and a smaller margin $t \times SE$.