Cholesky decomposition:

A symmetric positive definite

$Ax = b \longrightarrow x = A^{-1} b \longrightarrow$ but computing A^{-1} is $\overline{O(N^3)}$ if A is $N \times N$

complexity

$\underbrace{A = L L^T}$ where L is a lower triangular with positive diagonal elements

Cholesky's decomposition

$L = \begin{pmatrix} \times & >0 & (0) \end{pmatrix}$

Instead of computing A^{-1}, do:

$(L \underbrace{L^T x}_{= y} = b)$

1) $L y = b$ (easy since L is lower triangular)

2) $L^T x = y$ (easy since L^T is upper triangular)

(with Gauss elimination)

Message: It is easier to do Cholesky than matrix inversion

$\text{np.}\cancel{\text{inv}}(A) \longrightarrow \text{np.solve}(A, b)$

GP regression:

$$\mu^* = K^{*T} K^{-1} y$$

$$\Sigma^* = K^{**} - (K^*)^T K^{-1} K^*$$

1) $Km = y \to$ Cholesky

$\mu^* = K^{*T} m$

2) $K^* = (K_1^* | K_2^* | \dots | K_M^*)$

For $i = 1, \dots, M$, solve $K m_i = K_i^*$

Define $Q = (m_1 | m_2 | \dots | m_M)$

Then $\Sigma_1 = K^{**} - (K^*)^T Q$