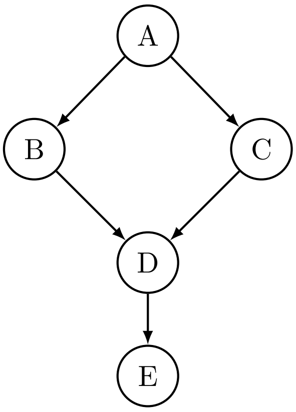
Quiz: Directed Graphical Models & Inference

STA414/2104 - Winter 2026

1. **Variable Elimination Complexity**
   Consider the following graph:



The joint distribution factorizes as:

$$p(A, B, C, D, E) = p(A)p(B|A)p(C|A)p(D|B, C)p(E|D).$$

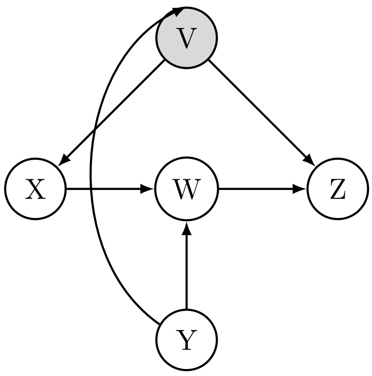
We want to compute the marginal probability $p(E)$. We choose the elimination ordering: **Eliminate A, then B, then C, then D**.

During this process, what is the maximum number of variables involved in a sum?

(a) 1
(b) 2
(c) 3
(d) 4

Correct Answer: (c)
The sum is $p(E) = \sum_D p(E|D) \sum_C \sum_B p(D|B, C) \sum_A p(A)p(B|A)p(C|A)$. *The last sum involves three variables and becomes* $\tau(B, C)$. *The next sum over* B *involves three variables and becomes* $\tau(D, C)$. *Then, the sum over* C *involves two variables and becomes* $\tau(D)$. *Finally, the last sum involves two variables (*E *and* D*).*

2. **d-Separation with New Variables**
Consider the following graph



We want to verify if $X \perp Y \mid V$ using the **Pruning / Edge Deletion algorithm**. Which of the following sequences correctly describes the steps of the algorithm?

(a) **Step 1:** No nodes can be pruned because W is a collider and Z is a descendant of V. **Step 2:** Delete edges outgoing from V. **Result:** Connected.

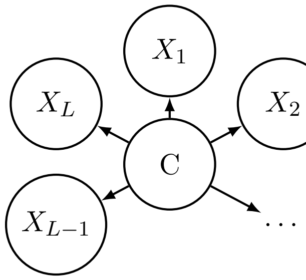
(b) **Step 1:** Delete Z. Then delete W (which becomes a leaf). **Step 2:** Delete the edge $V \to X$ (outgoing from evidence). **Result:** X and Y are disconnected.

(c) **Step 1:** Delete Z. W cannot be pruned because it connects X and Y. **Step 2:** Delete edges outgoing from V. **Result:** Connected via W.

(d) **Step 1:** Delete edges outgoing from V. **Step 2:** Prune W because it is separated from V. **Result:** Disconnected.

Correct Answer: (b)
Rationale: 1. ***Delete Nodes:*** *Z is a leaf and not in $\{X, Y, V\}$, so prune Z. Now W is a leaf (since $W \to Z$ is gone). W is not in $\{X, Y, V\}$, so prune W as well. 2.* ***Delete Outgoing Edges:*** *The evidence is V. Delete edges starting at V: delete $V \to X$ (and $V \to Z$, though Z is already gone). 3.* ***Check Connectivity:*** *The only remaining edge is $Y \to V$. X is isolated. Thus, they are disconnected (independent).*

3. **Variable Elimination on Star Graphs**
Refer to the complexity formula $\mathcal{O}(m \cdot k^{N_{max}})$. Suppose you have a "Star" graph where the central node C is connected to L leaf nodes $X_1, \ldots, X_L$.



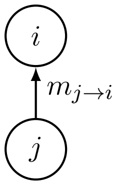
If you choose to **eliminate the center node C first** to compute $p(X_1)$, what is N_{max}?

(a) $N_{max} = 2$
(b) $N_{max} = L + 1$
(c) $N_{max} = L$
(d) $N_{max} = 1$

Correct Answer: (b)
Eliminating C requires multiplying all factors involving C. Since C is connected to all L leaves, the joint factor involves C and all X_i, totalling $L + 1$ variables:

$$p(X_1) = \sum_{X_2} \cdots \sum_{X_L} \sum_{C} p(C)p(X_1|C) \ldots p(X_L|C).$$

4. **Belief Propagation (Sum-Product)**
In Belief Propagation on a tree, we calculate the message $m_{j\to i}(x_i)$. Suppose node j is a **leaf node** and is **unobserved**.



(Leaf)

What is the content of the message $m_{j\to i}(x_i)$ sent to its parent i?

(a) It is exactly the factor $\psi_{ij}(x_i, x_j)$.
(b) It is a constant.
(c) It is $\sum_{x_j} \psi_j(x_j)\psi_{ij}(x_i, x_j)$.
(d) It is $\psi_j(\bar{x}_j)\psi_{ij}(x_i, \bar{x}_j)$.

Correct Answer: (c)
Because j is a leaf, the product over its children is empty (equals 1). The message is simply the marginalization of its local and edge factors.